SHRP 2 C04:
Improving Our Understanding of How Highway Congestion and Pricing Affect Travel Demand

APPENDIX B

SUPPORTING TECHNICAL MEMORANDA

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Table of Contents

APPENDIX B1

PSRC Traffic Choice Study Data Review and Completed Processing Tasks

MEMORANDUM

Subject:	PSRC Traffic Choice Study Data Review and Completed Processing Tasks
Prepared by:	Marcelo Oliveira, GeoStats
Submitted to:	Mark Bradley
Date:	03/30/2010

Introduction

GeoStats received two DVDs from Mark Bradley containing database dumps of both the raw GPS data (Oracle backup) and processed databases (PostgreSQL dumps) for the PSRC Traffic Choice Study. Following initial guidance provided by Mark Bradley, GeoStats focused on extracting the data from the processed clean database stored in the clean_backup.pgdump file. This extract was imported into a postgreSQL database instance on one GeoStats' development servers (columbus.geostats.com). The imported SQL file extracted from the archive was over seven gigabytes in size. Table 1 identifies the tables found in the database once it was loaded.

TABLE 1: TABLES, FIELDS AND INTERPRETATION OF CONTENTS FOR THE CLEAN PSRC DATABASE

Table Name	Fields	Interpretation of Contents
clean_actual_main_data	mdh_zip, mdh_city, mdv_type, obu_id, obu_serial_number, mvo_date_from, is_fieldtest, family_id	Basic data about the participating households and vehicles. (contains 473 records)
clean_daily_link_usage	link_date, link_usage, rds_id, rds_link_id, rds_short_name, rds_name, rdr_id, rdr_short_name, rdr_description	Summary daily usage by road link. (contains 1,557,461 records)
clean_daily_mileage	vmt_date, vmt_total, vmt_tolled, vmt_zero_tolled, vmt_non_tolled, daily_toll_amount	Total mileage statistics by day. (contains 145,331 records)
clean_final_account_values	bka_balance, bka_chng_ts, obu_id	Final account values by vehicle. (contains 473 records)
clean_monthly_account_summary	bk_month, bk_year, total_charges, obu_id	Monthly account summary information by vehicle. (contains 6,260 records)
clean_obu_gps_fix_leak	obu_serial_number, gps_fix_leak, obu_id	Unknown, might be related to vehicle equipment. (contains 450 records)
clean_obu_transmission_leak	obu_serial_number, transmission_leak, obu_id	Unknown, might be related to vehicle equipment. (contains 452 records)
clean_tolled_links	obu_id, mvo_date_from, mvo_date_to, bka_status, bk_create_ts, bk_gps_lon, bk_gps_lat, bk_gps_hdop, bk_bktp_trip_id, bk_ces_toll, rds_link_id, rds_short_name, rds_name	Individual link tolling records (contains 5,573,513 records)
clean_tracking_data	obu_id, mvo_date_from, bkt_gps_lon, bkt_gps_lat, bkt_gps_hdop, bkt_gps_ts, bkt_local_ts, bkt_bktp_trip_id	Logged vehicles GPS points (contains . 27,086,242 records)
clean_trips	obu_id, mvo_date_from, bktp_trip_id, bktp_start_date, bktp_end_date, bktp_mt_total, bktp_mt_tolled, bktp_mt_zero_tolled, bktp_mt_non_tolled, toll_amount	Processed trip records containing tolling summary statistics. (contains 761,174 records)

Initially, GeoStats extracted the records for the eight tables that have less than one-million records to a Microsoft Access database for review by other project team members. For this initial review GeoStats also generated an augmented version of the clean_trips table that included the start and end coordinates for each trip, brought in using a join with the clean_tracking_data table.

Completed Data Processing Tasks

GeoStats reviewed the memo entitled "Possible Uses for the Puget Sound Traffic Choice Study Data in the SHRP II C04 Project" and identified a set of data processing tasks. These tasks were grouped into a set of immediate ones that could be undertaken without a need for additional data and another group that would require additional information and supporting data files from PSRC. The following sections describe the work completed on these tasks.

Home / Work Location Identification Process

GeoStats implemented a processor that determined destination locations associated with households based on household vehicle trip ends. Unique locations were created using a clustering technique that naturally grouped trip end coordinate into unique locations. In order to avoid GPS cold start related problems only trip end coordinates were used to generate locations.

Home and work locations were determined based on the distribution of the amount of time spent at each location by dividing a day into four time periods. The time bins used were morning (0601-1200), afternoon (1201-1800), evening (1802-2400), and night (0001-0600). The time bin data was originally stored on the household vehicle level and then aggregated up to the household level. This allowed us to determine location types based on the household as a whole as well as locations on a per vehicle basis.

The first location type of interest was the home location. This location was defined as the location with the most hours in the night bin at the household level. The assumption was that most family vehicles would be home between midnight and six in the morning. Even if some members of the family work night shift jobs, there is a chance that the total from the other family members will allow the correct home location to be identified.

Work locations were determined for each vehicle based on the location where that vehicle spent the most hours in the morning and afternoon bins and that is not already defined as the home for the household. The total duration covered in these two bins indicates the normal work hours of the average first shift job, with some allowance for flexible start and end times.

Evening hours were not considered when determining the home or work locations because it is not possible to predict where a person would spend that time. This method may not properly identify locations for people who work unusual schedules. These outliers could be identified once typical vehicle schedules are complete.

Tour Analysis

Tours were defined as series of trips that start and end at predefined locations. Processing was done for home and the vehicle work based tours. A total of four tour types were identified: home to work, work to home, home to home, and work to work. The location information generated in the previous was used to generate tours.

Tours were identified by looping over a start time sorted list of all the trips for a vehicle. A new tour started when the home or work location was identified as the trip start location. The tour continued until the home or work is identified as a trip end location. Once start and end points of each tour were identified the tour type was determined.

Link Analysis

GeoStats performed additional processing on the tolled link data with the objective of relating link start and end nodes to the PSRC model's traffic analysis zones (TAZ) and also to compute time spent on each link. In order to complete this task the PSRC model network was imported into the postgreSQL database storing the experiment data and its node identifiers were mapped to the link identifiers used in the clean_tolled_links table. A simple C# tool was then developed to process the vehicle GPS points with respect to the records captured in the clean_tolled_links table.

However, reasonable link travel times could not be estimated for a sizable proportion of the captured tolled links. This was mostly because of the coarse nature of the used model network's geometries, which resulted in large deviations between the trajectory of subsequent GPS points and the links, especially in longer model links.

Furthermore, the entry times reported in the clean_tolled_links table did not match the timestamps of the GPS points closest to link origins leading the team to believe that some additional processing was done to determine these times. Also, the time resolution of the estimated exit times was determined by rather low logging frequency used to collect the original data (between 0.1 and 0.033 Hz). These two factors combined resulted in inaccurate link travel time estimates. Given this, it was decided that this analysis should not be pursued any further.

Parcel Analysis

GeoStats imported parcel data from PSRC into the same database used to store the traffic choice data. A spatial analysis that matched locations to parcels was performed using a point in polygon operation in PostGIS (spatial extension for PostgreSQL). If a location did was not covered by any parcel, a buffer distance of 250 meters was used to determine the nearest parcel which was then was associated with the end location. This last step was necessary because vehicles may park in public areas which are typically outside land parcels.

Typical Schedule Analysis

Typical schedules were identified on a household vehicle level and were based on all tours for each vehicle. Tours were identified and classified by analyzing vehicle trips. The day of the week for each tour was identified and the tour was added to that day of the week's schedule. Once all vehicle tours were assigned a typical vehicle schedule was generated.

Vehicle schedules were created on a per-day basis using a Quality Threshold (QT) clustering algorithm. QT clustering is a recursive process where each data point is considered as the center of a cluster; then all remaining points are compared to the cluster center to determine which ones fall into a cluster of maximum size located at that center point. The cluster with the largest number of points is saved, all points inside that cluster are removed from consideration, and the process is repeated with the remaining data points until no points remain unassigned to a cluster.

In this case, the cluster size was set not to exceed thirty minutes, which allowed for a fifteen minute buffer on each side of the average start time for example. Each cluster was considered a schedule event and stored in the database. The event time was computed as the average start time of all tours contained in the event. The usual destination of the event was represented as the tour destination that occurred with the most frequency in the event tours.

A schedule was created for each day of the week that had tours associates with it. No consideration was given to holidays or other events that could cause a shift in the normal daily activities. The assumption for this was that the most frequently occurring tours would prevent these outliers from changing the schedule in a significant way.

Data Deliverables

In addition to the initial Microsoft Access database, GeoStats delivered incremental results from the above mentioned data processing exercises to Mark Bradley in a series of comma-separated text files and Excel workbooks.

APPENDIX B2

Processing of Traffic Choices GPS Data to Provide Measures of Travel Time Variability For SHRP 2 C04

Processing of Traffic Choices GPS Data to Provide Measures of Travel Time Variability for SHRP 2 C04

Mark Bradley

March 20, 2009

With the assistance of GeoStats, we will be processing the GPS data to obtain some estimates of day to day travel time variability within specific time of day periods, to use as an explanatory variable in disaggregate model estimation. One approach would be to use travel times between the origins and destinations of actual trips that are repeated on different days. Two difficulties with that approach are (a) there are so many trip origin and destination points that there would be very few observations per pair, and (b) some of the actual travel time for the trip is made on local streets that are not modeled network links.

We can avoid both difficulties above by taking as the trip "origin" and "destination" the entry points to the first and last modeled network links used during the trip. For example, if a an actual trip traverses 10 links, then the time entering the first link would be used as the "trip" start time, and the time entering the tenth link would be used as the "trip" end time. Another advantage of that approach is that each link pair along the actual trip could be treated as separate pseudo-trip—link 1 to link 2, link 1 to link 3, link 1 to link 4, link 2 to link 3, etc.—so there are many more observations. (This could present possible "repeated measures" issues when using the data in analysis, but that should not be an issue when analyzing any particular link pair.)

Some exploratory GPS data analysis has been completed to look at the feasibility of the above approach. Here are some summary statistics from the analysis thus far:

- There are 297 households in the sample with trip records
- There are 448 vehicles in the sample with trip records. That is an average of 1.51 cars per hh.
- There are 693,855 trip records in the sample with valid travel time information
- That is 2,336 trips per household on average during the study period. The minimum is 96 trips, and the maximum is 7,100 trips.
- That is 1,549 trips per vehicle on average during the study period. The minimum is 3 trips, and the maximum is 3,853 trips.
- There are 5,573,513 records for network (tolled) links traversed during the trips. That is an average of 8.03 used links per trip.
- Across all trips, 6,421 different network links were used.
- Each link was used during 868 trips, on average. The minimum is 1 trip, and the maximum is 13,919 trips.
- 1,508 links were used on 1,000 or more trips
- 2,702 links were used on 500 or more trips
- 4,891 links were used on 100 or more trips
- In all, there are 41,222,820 possible directional link pairs (6,421 x 6,420).

- Of those, 2,140,319 link pairs were actually traversed during weekday trips (5.2% all of possible pairs)
- Each link pair was used during 18 trips, on average. The minimum is 1 trip, and the maximum is 9,271 trips.
- 4,845 link pairs were used on 1,000 or more trips
- 11,983 link pairs were used on 500 or more trips
- 66,014 link pairs wee used on 100 or more trips
- Selecting just the 66,014 link pairs with 100 or more trips, there are 50,233,094 link pair "trip" records, an average of travel time 761 observations per link pair.

The file noted in the last bullet point will serve as the basis for further analysis. 50 million "trip" records should be sufficient to generate a good deal of information on variability. Each record contains:

- ID of the "origin" link
- ID of the "destination" link
- Day and time entering origin link
- Day and time entering destination link

We can use this information to generate travel time distributions for each link pair/time period of day combination, and then somehow relate those back to the zone-to-zone skims for model estimation. There are a number of ways that this could be done. Below are some questions for people to provide input on:

- What time period resolution would be good for this analysis. Full hours? Half-hours during the peaks?
- What it the minimum number of observations per link-pair / time period necessary to give a reliable distribution?
- How should the results be aggregated geographically to values that can be used with TAZ-to-TAZ skims? One method would be to first aggregate links to zones and then do the analysis at the TAZ-to-TAZ level (probably eliminating intra-zonal link pairs). Another method would be to first get distributions at the link pair level and then somehow generalize that to zone-to-zone (not sure exactly what that would look like).
- How can this method be applied in future scenarios when GPS data is not available? To do that, we might want to use the results to estimate a regression model that predicts, say, the ratio between the median and the 80% or 90% travel time as function of other variables. Some suggestions for the variables:
    - The ratio of the median travel time to the free flow travel time (maybe a non-linear function)
    - The fraction of the total path distance that is on various facility types, such as freeways, bridges, etc.

    - The time period within the peak (early shoulder, late shoulder, etc.) relative to the directionality (toward the CBD, away from the CBD) and the location (inner suburbs, outer suburbs, rural, etc.)
    - Others?

The analysis could be done either at the link-to-link or zone-to-zone level.

APPENDIX B3

Possible Uses of the Puget Sound Traffic Choice Study Data in the SHRP II C04 Project

Possible Uses for the Puget Sound Traffic Choice Study Data in the SHRP II C04 Project

Mark Bradley

September 15, 2008

Introduction

The Traffic Choices Study was a unique behavioral experiment carried out by Puget Sound Regional Council (PSRC) for the Federal Highway Administration (FHWA) Value Pricing Pilot Program. In the study, selected Seattle region households reacted to variations in toll level by road type and time of day over an 18 month period, with in-vehicle GPS units used to record behavior as accurately as possible, and also to keep track of what toll fees to charge the respondents. The information in this memorandum is based primarily on two PSRC documents: "Traffic Choices Study: Summary Report", from April 2008, and Appendix 19 to that report "Traffic Choices Study: Toll Impact Models". I have received from PSRC the base GPS trace data in Oracle export format, as well as a cleaned relational database in PostgreSQL dump format. I have not yet been able to open those files for analysis, although I have downloaded PostgreSQL software and made some initial attempts at using it.

Description of the Experiment

The Traffic Choices Study combines some of the best features of Revealed Preference (RP) and Stated Preference (SP) data collection approaches. Similar to SP, the experiment was able to obtain behavioral responses to a policy that has not yet been implemented—namely ubiquitous, mileage-based congestion pricing on all freeways and main arterials in the Seattle region. In this case, however, the study was able to overcome the hypothetical nature of SP methods by applying the pricing during real trips that the respondents made over an extended period of time and charging those respondents real money for using specific roads at specific times of day and week. This was done using the innovative approach of giving respondents a fixed sum of money in an account at the beginning of the experiment, given respondents a toll map and schedule to inform them what when and where tolls would be levied against that account, and then using in-vehicle GPS to determine what level of per-mile toll applied at any instant the vehicle was being used and relaying that information to the driver. At the end of the experimental period, respondents were allowed to keep whatever funds remained in their account. This system mimicked as closely as possible the way that funds are charged against the credit cards of users of actual electronic tolling systems. The main differences compared to an actual congestion pricing system were that (a) only a small subpopulation of all drivers on the roads were faced with the experimental pricing, so there was no noticeable effect of pricing on overall traffic levels or congestion, and (b) respondents were spending money that was given to them as part of the experiments, and some may not have felt as if they were spending "their own" money. Later paragraphs in this memo discuss what implications these differences may have for behavioral modeling.

For the study, GPS units were installed in all household vehicles in 275 randomly recruited households in the region, providing a sample of more than 400 instrumented vehicles. Before tolling was "turned on", respondents drove with the GPS units in their vehicles for a period of three months (see timeline in Figure 1). This initial non-priced period served a few different purposes: (a) to make sure that the GPS units were working and transmitting data properly to the central facility, (b) to collect "baseline" behavioral data against which data from the tolled situation could be compared in analysis, and (c) to get an idea of how many miles each household regularly drove on the tolled links, so that the initial funding level of the user account could be set. (The objective was to set the budget high enough so that users would not fully deplete the account and have to leave the pricing experiment early, while at the same time not setting it so high that some households would still receive a significant "reward" at the end even if they did not adjust their trips to avoid paying the tolls.)

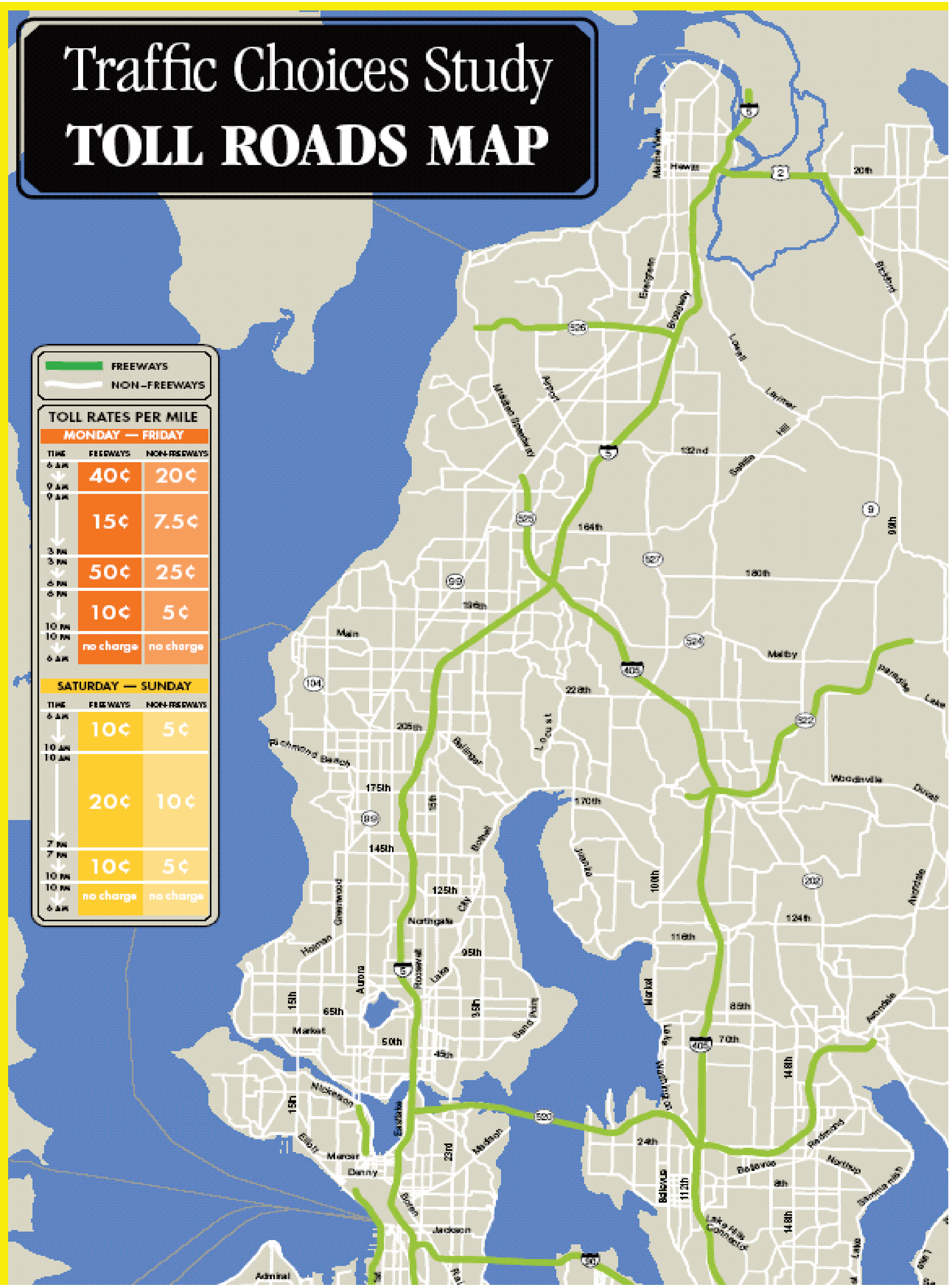
Figure 2 is the "Toll Roads Map" that was given to respondents to study at home and also keep in their vehicles. The map shows two types of road that were priced: the main freeways shown in green, and other main arterials shown in white. The toll rates per mile for freeways were set twice as high as for the other arterials, ranging from 10 cents to 50 cents per mile on weekdays and 10 cents to 20 cents per mile on weekends, varying by time of day. On weekdays, the highest priced period was the PM peak from 4 pm to 7 pm, followed by the AM peak from 6 am to 9 am. Prices were lower midday and in the evening (7 to 10 pm). On weekends, the high toll period was 10 am to 7 pm. No tolls were charged between 10 pm and 6 am on any day. All respondents received the same toll schedule for the entire experiment –no variation was used across the sample or across seasons/months.

The pricing was operational beginning on July 1, 2005 and continued through February, 2006, a period of 8 months. During that time, respondents could obtain information in their vehicle as to what was being charged at that moment, and also in total for that trip or that day. Respondents could also go on-line to the project website to get a historical overview of what toll roads they had used and when, what tolls they had been charged, and how much money remained in their account. During the total project period, across the sample, the GPS units logged over 750,000 individual trips, including over 100,000 toll transactions. The central system also sent out over 4,000 customer billing invoices, mimicking the type of monthly invoice that would be sent in an actual system. A further month or so of control data was collected after tolling ended.

Figure 1: Traffic Choices Project Timeline

<table>
<tr><td rowspan="3">Pre-implement ation</td><td colspan="9">2005</td><td colspan="6">2006</td></tr>
<tr><td>A</td><td>M</td><td>J</td><td>J</td><td>A</td><td>S</td><td>O</td><td>N</td><td>D</td><td>J</td><td>F</td><td>M</td><td>A</td><td>M</td><td>J</td></tr>
<tr><td colspan="3">Control</td><td colspan="8">Experimental Treatment - Tolls</td><td>Control</td><td colspan="3">Analysis →</td></tr>
</table>

Figure


Traffic Choices Study
TOLL ROADS MAP
FREEWAYS
NON–FREEWAYS
TOLL RATES PER MILE
MONDAY — FRIDAY
TIME FREEWAYS NON-FREEWAYS
6 AM – 9 AM 40¢ 20¢
9 AM – 3 PM 15¢ 7.5¢
3 PM – 6 PM 50¢ 25¢
6 PM – 10 PM 10¢ 5¢
10 PM – 6 AM no charge no charge
SATURDAY — SUNDAY
TIME FREEWAYS NON-FREEWAYS
6 AM – 10 AM 10¢ 5¢
10 AM – 7 PM 20¢ 10¢
7 PM – 10 PM 10¢ 5¢
10 PM – 6 AM no charge no charge
Woodinville
Bellevue
Redmond
Malby
Richmond Beach

2

Previous behavioral analyses

Some behavior analysis has already been carried out by PSRC and EcoNorthwest, and reported in the study report. This section provides a brief summary of the findings. The majority of the analyses have been done at a fairly aggregate level where the unit of analysis is not a particular trip or route, but rather the average travel per week during the tolling period versus during the control period. One reason for a more aggregate level of analysis is a data issue particular to GPS data, which any analysis of the data must deal with. The issue is that all of the data is passive and vehicle-based. As a result, for any particular trip the data is missing three items of information that are often used in analysis of household travel survey data: (1) which person in the household is driving the vehicle, (2) how many occupants are in the vehicle, and (3) what type/purpose of activities are carried out at each stop location. The initial analyses have addressed this issue to some extent by identifying the location of regularly-visited workplaces. With this information, all tours (or partial tours) could be categorized into four types—home to work, work to home, home to home (non-work tours), and work to work (work-based subtours). Also, analyses were performed at three levels of aggregation: each household, each vehicle, and each workplace.

Overall, compared to the control period, introduction of the tolls was found to produce the following impacts on travel patterns across all participating households:

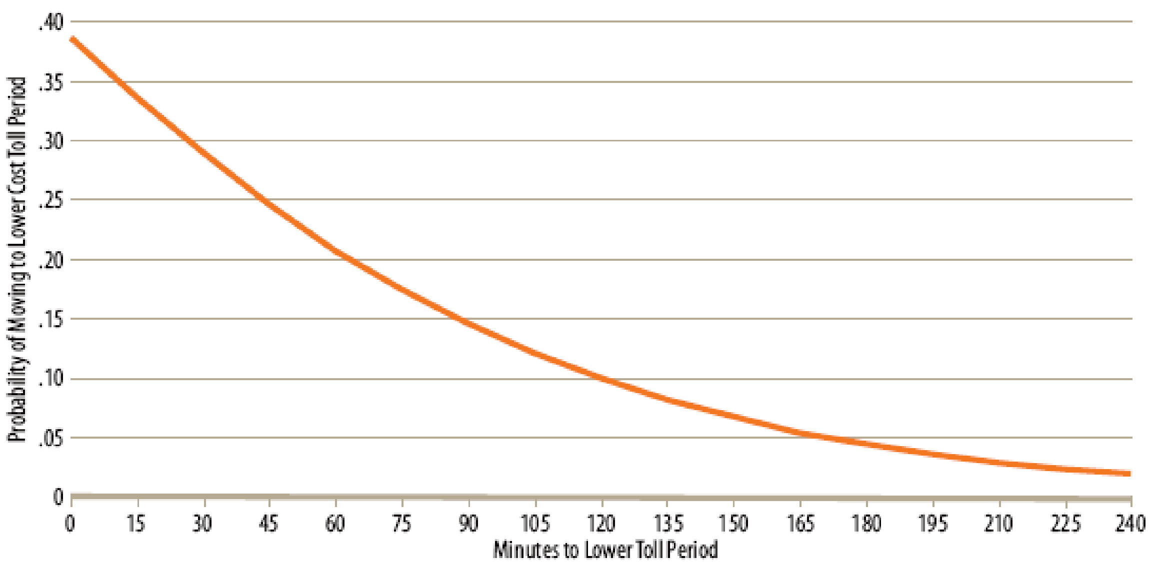
- 7 percent reduction in all vehicle tours (tours per week)
- 6 percent reduction in tour segments (segments of tours per week)
- 8 percent reduction in tour drive time (minutes of driving per week)
- 12 percent reduction in vehicle miles traveled (miles per week)
- 13 percent reduction in miles driven on tolled roads (tolled miles per week)

From these numbers, we can infer a number of behavioral findings:

- In the “big picture”, the tolling had enough of an effect on various dimensions of behavior that the data should be suitable for further analysis on the effects of pricing and congestion.
- The fact that tour segments (trips) were reduced slightly less than the number of tours implies a slight increase in trip chaining—i.e. the number of trips per tour increased by about 1 percent.
- The fact that vehicle miles traveled decreased by 12 percent while the number of tours decreased by only 7 percent implies a reduction in average tour distance by about 5 percent. This could be because longer distance tours were most likely to be suppressed, but it could also be due to destination switching toward closer destinations. A comparison of tour distance distributions with and without tolling would provide further insight.
- The fact that vehicle miles traveled on toll roads decreased slightly more than vehicle miles travelled overall implies at least a small amount of shifting from tolled routes to non-tolled routes and/or to the non-tolled night period (although this comparison does not identify route shifting to routes and/or times of day that are still tolled but at a lower toll level).
- The fact that total travel distance decreased by 12 percent but the total drive time decreased by only 8 percent implies that overall average driving speed is reduced by about 4 percent. This is likely due to the fact that there is less driving on the tolled freeways, which have the highest speeds.

An additional analysis was carried out to look at departure time shifts for home-to-work journeys. From the travel patterns in the control period data, it was possible to determine the usual departure time from home to work for the majority of regular commuters. Then, an analysis was done to relate the percentage of those commuters who shifted to a lower toll period as a function of the number of minutes the departure time had to be shifted away from the usual time in order to move into the lower toll period. The reported results are shown in Figure 8 from the PSRC report (below), with a clear relationship showing over 30% of commuters shifting time when the required shift was 30 minutes or less, down to less than 10% shifting when the required shift was more than two hours.

Figure 8. Home-to-Work Tour Probability of Moving to Lower Toll



Although it is difficult to provide much interpretation of these results without knowing more detail about the analysis, below are some implications of these findings in the context of further analysis of the data:

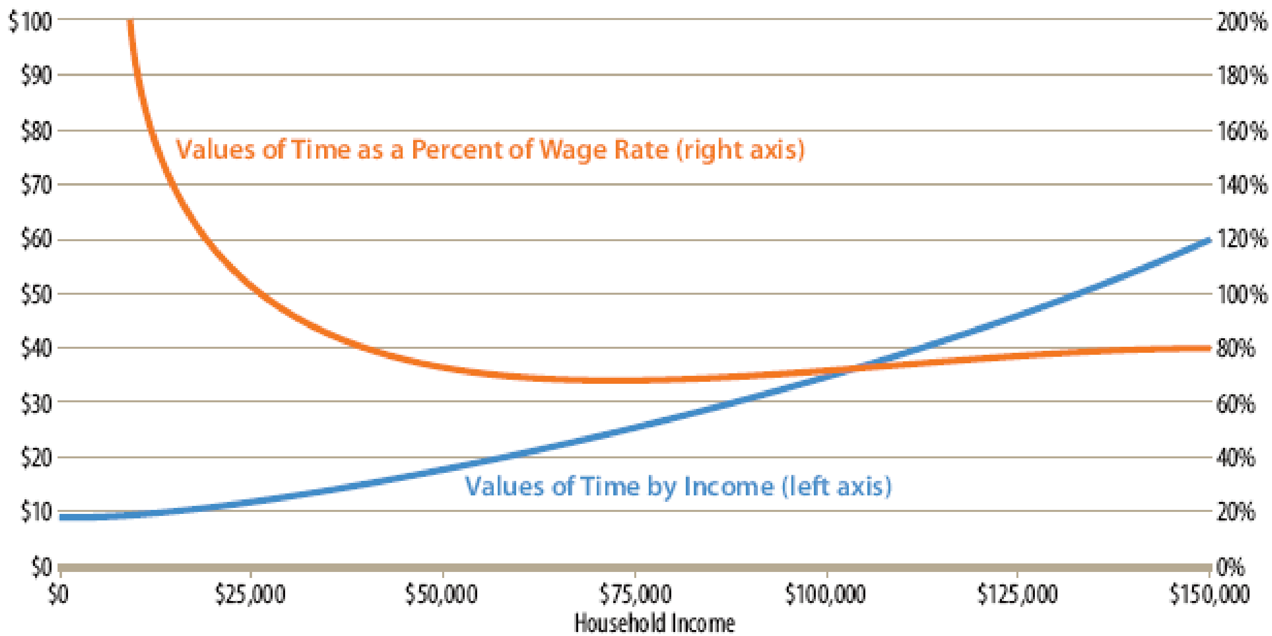
- There appears to be enough systematic departure time shifting in the data to support disaggregate departure time modeling, at least for home to work journeys. It is likely that work to home journeys could be analyzed in a similar way, preferably jointly with the home to work journey.
- It is possible that the data could be analyzed to find other regular non-work journeys that particular households make during both the control and tolling periods. Home to school/university tours and tours to escort children to school seem likely candidates, and school locations would be fairly easy to pinpoint in the data by matching to a GIS parcel database, although school start and end times are typically fixed and home-school distances are typically short, and thus not a good candidate for departure time shifting to avoid tolls. Perhaps there are other types of regular journeys as well, although inferring what the destination purpose of the journeys was would require GIS analysis.
- A multivariate analysis approach would provide more useful behavioral models, including not only the amount of time shift necessary, but also the direction of shift, the difference in toll levels and

travel times between the periods, and other characteristics of the household, the driver (if known), the destination, and the tour.

- People may shift their route of travel instead of (or in addition to) their departure time. An advantage of GPS data over typical household survey data is that the exact route of travel can be identified. With the data available, it seems that a joint route-departure time choice model would be a more complete and valid way of identifying the simultaneous effects of pricing, travel time and congestion.

Route choices for home to work journeys were also analyzed by the PSRC project team. Identified in the data was the percentage of times that each commuter chose to use an alternative lower toll or non-tolled route, and that was analyzed with respect to the toll difference and travel time difference between the routes, in order to infer a value of time (VOT) for each commuter. The results were then interpreted as a function of household income, with the resulting VOT function shown below in Figure 9 from the PSRC report. Except for the very low income households, the imputed VOT appears to be 70% to 80% of the wage rate across the full range of incomes. These are somewhat higher than VOT typically estimated from SP data on route choice under pricing, although in line with typical VOT from RP data.

Figure 9. Observed Home-to-Work Tour Values of Time (As a Function of Route Choice)



Again, it is difficult to provide an interpretation or critique of these results without knowing more detail about the analysis method. Below, however, is a discussion in the context of further analysis of the data:

- Shifting route is only one possible way to reduce or avoid paying tolls. Shifting departure time is the other most likely way, but other possibilities include shifting mode to transit or non-motorized (this would only be seen in the GPS data as a reduction of commute frequency), increasing car occupancy (this would not be seen in the GPS data at all), shifting destination (very unlikely for work tours, at least in the short term), and canceling commute trips, e.g. by telecommuting (in the GPS data, this would be indistinguishable from switching mode). Since these other shifts would most likely be made by people with the lowest VOT (highest marginal disutility of toll and/or lowest marginal disutility of travel time), the fact that those cases are not in the route choice data may lead to a higher imputed VOT than is the case in general. If it is true, however, that route choice is truly the "lowest level" choice in the decision hierarchy, then the high VOT may be suitable for the particular context of route choice.

- As stated above in the context of departure time choice, the best way to sort out these issues is with a joint model that includes the three main identifiable dimensions of commuting behavior in the data set—route choice, departure time choice, and frequency of commuting by car—and analyzes them in an integrated manner, including as many household, person, land use, and contextual variables as possible. The final section lays out a methodology for achieving this.

Proposed discrete choice analysis for auto commute tours

For regular home to work tours that can be identified in the data, I propose the following discrete choice model specification for a joint route choice/departure time choice/frequency choice model:

<u>Time of day choice alternatives</u>

In the simplest case, I propose using five time perioids:

1. Night/early 10 pm – 6 am
2. AM peak 6 am – 9 am
3. Midday 9 am – 4 pm
4. PM peak 4 pm – 7 pm
5. Evening 7 pm – 10 pm

These are the same as the different toll periods offered during the Traffic Choices Study, so they will capture the time-of-day variation in the experimental toll levels. Also, they are the five time periods used in the PSRC regional model, so travel time skims using estimates of congested network link speeds are already available for each time period for 2006.

Since the assumption will be that all of the toll links driven in a half-tour are within the same tolling period, it may be best to offset the periods used to define the alternatives a bit from the actual tolling periods. For example, if someone arrives at work at 6:05 am, it is likely that they actually drove on most of the tolled links during the no-toll period before 6 am rather than during the AM peak tolling period. So, giving a 15 minute leeway would say that the Night/early arrival period would go until 6:15 am, the AM peak arrival period would go until 9:15 am, etc. For departure from work, it would go the other way—the PM peak departure period would begin at 3:45 pm instead of 4 pm, etc.

At the tour level, if we screen out any work tours that go overnight past 6 am, then there are 15 possible combinations of the 5 arrival periods at the workplace and the 5 departure periods from the workplace. PSRC also has a peak-spreading model which breaks down each 3-hour peak period into 6 half-hour slices, and can also provide some travel time skims for those half-hour periods. It would be possible to expand the model to include those as separate TOD alternatives as well. There would not be any variation in experimental toll level between the half-hours in a peak period, but there would be variations in travel times (and perhaps in reliability as well, if we can obtain proxy data).

Route choice alternatives

There are dozens of different routes a person could to get from one point to another. However, most of those routes would be clearly inferior and unlikely to be chosen. For this model, I propose using a choice set of four possible routes:

1. The actual chosen route, determined from the GPS data
2. The fastest non-chosen route that uses at least one freeway link
3. The fastest non-chosen route that uses no freeway links, but uses at least one tolled link
4. The fastest non-chosen route that uses no tolled links at all

There are several things to consider when deciding on the best modeling strategy for route type choice. For one thing, several of the work tours in the data will contain intermediate stops. For this particular model, it is probably best to assume that the number and location of intermediate stops is fixed, so the various routes in the choice set need to visit all stops. That means that the choice attributes would have to be determined not just for each home-workplace combination in the data, but for each separate tour.

Also, the determination of the fastest alternative route could depend on the time of day. It would likely be too complicated to determine a different set of route alternative for each of the time period combinations. A simpler approach would be to just use the AM peak link speeds to determine the set of route alternatives for the home-to-work tour half, and use PM peak link speeds to determine the best set of routes for the work-to-home tour half.

The process of finding the set of best alternative routes can be done with standard network route shortest path software, with at least two potential complications: First, it should be fairly easy to find the fastest route that uses no freeway link by just setting all the freeway links to very slow speeds. Similarly, the fastest route using no toll links can be found by setting all tolled links to very slow speeds. The complication may be if one of those routes is identical to the actual chosen route for both half tours. In that case, two options would be to (a) use the second fastest path instead for at least one of the half-tours, or else (b) if the software cannot be made to report the second fastest path, just set that particular route alternative to be not available, since it is already in the choice set.

Second, the standard PSRC regional model network may not be detailed enough at the sub-arterial level to identify actually chosen paths, or to even find any path that uses no tolled links. The ideal solution would be to use a network that is augmented with an overlay of the local street network (perhaps this would be more easily done within GIS software than with network modeling software). If that is not possible, a shortcut would be to just use whatever sub-arterial streets are in the modeled network as an approximation to the travel time on the full sub-arterial network, and to not include sub-arterial streets as matching criteria between the actual routes and the modeled routes—in other words, if the actual path and network path match along all arterials and freeways, then they are a match. This decision would only be very important for the 4th route type (using no tolled links at all), and that type of route alternative is probably very inferior for longer trips in any event.

Commute frequency alternatives

This choice dimension could seen as the most "optional" and speculative of the three, but it may be possible to add some useful results to the modeling process for very little additional effort. There are only two choice alternatives here:

1. Commute to the regular workplace by household vehicle on a given day.
2. Do not commute to the regular workplace by car on that day.

Note that the latter alternative could be one of a few different things—(a) the person went to the regular workplace, but used some other mode or went in a vehicle that was not one of the household's vehicles; (b) the person went to a different workplace on that day, such as to a meeting; (c) the person stayed home from work that day, to telecommute, because they were sick, or for some other reason. It is not possible in the GPS data to determine which of these was the actual case—all the data can tell us for sure is whether or not any of the household's vehicles went between home and the usual workplace that day.

To model commute frequency, we need to infer any days on which a commuter could have driven to work but did not. The simplest method is to assume that each commuter works on each weekday, so on any non-holiday weekday that there is no home-work tour in the GPS data, they are modeled as having chosen alternative 2 above. It is not so important whether or not the person actually worked five days per week or not—rather the important thing is the difference in commute frequency for that person during the toll period relative to the control period, and that is captured by this approach.

When we infer a non-commute day, we also need to assume whether or not the person would have made any intermediate stops had they gone to work. The simplest assumption would to assume no stops, so the various route choice alternatives would include paths directly to work and back. (In fact, a further simplification would be to use that assumption for all work tours. That would be analogous to tour-level models in a typical activity-based model system where we often do not model the number or location of intermediate stops until after the tour level models are applied. In that case, however, we would obtain an inferior route choice model, because in many cases we would be modeling the route connecting a sequence of locations that is different from the sequence of locations actually visited.)

Putting the three choice dimensions together, the recommended structure would have 61 possible choice alternatives—4 route types for each of 15 work arrival period/departure period combinations, plus the additional "null" alternative of not driving to the usual workplace that day.

As mentioned above, a way of getting more detail into the model for time of day choice is to use shorter periods—typically periods of either 30 minutes or 60 minutes duration. With four route types for each time of day combination, using such short time period alternatives would lead to a joint model with very many alternatives. In that case, three possible ways of simplifying the model would be: (a) only use the shorter periods within the AM peak and PM peak, and otherwise use the longer periods for Midday, Evening and Night; (b) model departure time choice and route choice sequentially instead of simultaneously, with route choice conditional on time of day, and route choice model inclusive value logsums included in the time of day model, or (c) model the home-to-work half tour separately from the work-to-home half tour. In the latter case, the work-to-home half-tour departure time could be modeled conditional on the work arrival time, so duration of stay at work could still be included as a variable in at least one of the models. Since the route choices are also separable by half-tour, each half tour model could be a joint route/departure time/frequency model.

Possible analyses for other tour purposes

As discussed earlier, it would be possible to carry out a similar modeling exercise for other travel purposes besides commuting. Any non-work tours to grade school locations would probably not be worthwhile for modeling due to the predominant use of local roads for such trips and the rigid start and end times for schools. The tolls would not be likely to have much influence on school trips. A possible exception is that of school tours made by university students, which could perhaps be combined into the commute analysis above.

Given that it is likely to be easy to identify school locations in the GPS data using the PSRC parcel-based land use database, it may be worth flagging such tours if only just to put them aside. That would leave the remaining non-work/non-school, or "non-mandatory" tours to model separately. Compared to modeling commute tours, non-mandatory tours will have the following characteristics that will affect how they are modeled:

- There are very relatively few regular destinations that are visited repeatedly at consistent times of day and week. So, the concept of a frequency model for a particular type of trip is not likely to be useful, nor is the concept of a "usual departure time" against which shifts can be modeled.
- The concept of a "half tour" is no longer useful, since many tours contain multiple destinations, with no clear way of determining a primary destination (particularly if no GIS land use analysis is done to determine the location type for each stop (shopping mall, residence, bank, etc.)
- Shifting the choice of destination(s) now becomes a relevant choice dimension.

If choice modeling is done at the tour level for non-mandatory tours, below are some specific recommendations for how it could be done, with the description in contrast to what is described above for commute tours:

Time of day choice alternatives

To be most useful, the definition of the time periods and time period combinations should be the same as for the commute models. In this case, however, since there is no particular anchor destination, the chosen alternative should be based on the departure period from home and the arrival period back home (in other words, home is used as the anchor location).

Route choice alternatives

The same four route types would be used as for the commute model, with the same definitions. Here, since there is no clear concept of a half tour, the route choice paths would need to be tour-specific for the specific series of stop locations visited on the tour.

Since the “null” alternative for tour frequency is not relevant here, the basic specification of this joint time of day/route choice model would be same as for the commute model, but without the extra “null” alternative and without a “usual departure time” to compare each time of day alternative against.

Destination choice?

It is interesting to think about how a destination choice dimension could possibly be added as a third choice dimension to this model. One possibility is to pre-define a number of distance bands, and randomly select one or more specific point destinations within each distance band to use as alternative destinations. A difficulty with this approach is what to do for tours with multiple destinations. Also, the random choice of alternatives is so arbitrary as to be meaningless unless a large number of destinations alternatives are specified, and that would be computationally infeasible within this joint structure.

A somewhat better option would be to match all trip ends to the PSRC parcel database to determine the land use type/building type for each parcel. Then, when designating alternative destinations within each distance band, one could only allow point locations that fall on the same types of parcels in terms of the type of land use (and possibly building size category). Again, the process would be complicated with multiple destinations, but with fewer degrees of freedom, perhaps an acceptable process could be designed (e.g. try to keep intermediate stops on the way to the most distant stop either at the actual location, or at a detour distance no greater than the actual detour distance).

A third option that is probably most feasible would be to model destination choice sequentially, using a more standard zone-based model specification with size variables, and including logsums from at least the time of day/route type choice utilities, calculated using zone-to-zone travel time and toll skims.

Data availability / formats

As mentioned above, we have the Traffic Choices data, but quite a bit of processing will still need to be done. One of the first tasks would be to create a trip file for all trips in the data, with the following items:

- Household id (link to a file with household and person characteristics for all HH members)
- Vehicle id (link to a file with vehicle characteristics, particularly fuel efficiency, and any information available on which HH member usually drives the vehicle)
- Trip day/month/year
- Trip day of week
- Trip origin departure time
- Trip origin location type (home, work, school, other)
- Trip origin XY coordinates
- Trip origin TAZ
- Trip destination arrival time
- Trip destination location type (home, work, school, other)
- Trip destination XY coordinates
- Trip destination TAZ
- Vehicle occupancy (if known)
- For each priced link/facility used during the trip…
    - The link/facility ID
    - The link/facility type (this, along with day of week and time of day should allow us to determine price/mile)
    - Distance driven on the link/facility

Once this basic data is able to be worked with, it can be further analyzed to process into tours, compare across days and months for regular commute tours, link to LOS/land use data for model estimation, etc. Thinking about LOS data, it would be ideal to have data on travel time variability for as many OD pairs and time of day periods as possible for 2006. The Seattle region has an extensive network of automatic traffic flow/speed monitors on the major freeway links, so that data could be useful here. Because travel time variations are highly correlated across adjacent links in a network, one cannot simply sum the variability across the links in a path, so measures need to be derived at the OD level – perhaps separately for freeway and non-freeway path types. The Traffic Choices data also has GPS data for individuals traveling on the same paths across different days, so some measures could also be derived from that data. The initial choice modeling can be done without these measures at the same time as such measures are being compiled/created. Such measures can also be used for modeling with the other Puget Sound RP and SP data sets from 2006.

In terms of who could/should carry out the processing of the GPS data and supply data, two main options are (a) Geostats, who are on our team and (b) PSRC, who are not on the team, but will find the model results and data useful for other projects (including the activity-based model project I am involved in), and may be interested in being co-authors on interesting papers and analyses that are produced from the data.

APPENDIX B4

Mathematical Formulations of the Integrated Multidimensional Network Choice Model

4. Mathematical Formulations of the Integrated Multidimensional Network Choice Model

In light of the section on the modeling framework, the integrated multidimensional network choice model includes two main sub models, namely time-dependent mode choice stochastic user equilibrium (TDMSUE) model, and multi-class multi-criterion dynamic user equilibrium (MDUE) route choice model.

Time-Dependent Mode Choice Stochastic User Equilibrium Model

Based on the weak law of large numbers, a mode choice probability $p_m^{wt}(y)$ in Eq. (1) can be obtained through mode flow y_m^{wt} divided by total OD demand, q^{wt}, as shown as follows:

$$p_m^{wt}(y) = \frac{y_m^{wt}}{q^{wt}}, \forall m \in M, w \in W, t \in T \tag{A1}$$

The TDMSUE conditions can be stated mathematically as follows:

$$y_m^{wt} = q^{wt} \times p_m^{wt}(y), \forall m \in M, w \in W, t \in T \tag{A2}$$

Therefore, the TDMSUE problem of interest can be formulated as the following fixed point problem. Let $\Omega(y)$ be a feasible set of mode flows.

Find $y^* \in \Omega(y)$, satisfying $y^* = q \times p(y^*)$. (A3)

Solving the above system of nonlinear equations will give a set of mode flows y^*, which is also the solution of the TDMSUE problem, i.e. y^* would satisfy the TDMSUE condition in Eq.(A2). To solve this problem by utilizing advanced optimization based, we reformulate this problem as a gap function based nonlinear programming (NLP) in Eq. (A4-a,b,c), which was proposed in a generalized dynamic stochastic use equilibrium problem (A3).

Min $g_M(y) = \frac{1}{2} \times \sum_w \sum_t \sum_m \left[y_m^{wt} - q^{wt} \times p_m^{wt}(y)\right]^2$ (A4-a)

s.t.

$$\sum_m y_m^{wt} = q^{wt}, \forall w \in W, t \in T \tag{A4-b}$$

$$y_m^{wt} \geq 0,, \forall w \in W, t \in T, m \in M \tag{A4-c}$$

where, objective function in Eq.(A4-a) is a gap measure defined by summation of square difference between assign mode flow, y_m^{wt}, and expected mode flow, $q^{wt} \times p_m^{wt}(y)$ over all OD pairs, departure times, and modes. Constraints in Eq. (A4-b) are flow balance constraints for each OD pair and departure time. Constraints in Eq. (A4-c) are non-negative mode flow constraints.

Multi-Criterion Dynamic User Equilibrium Route Choice Model

Based on the MDUE definition, the MDUE conditions can be mathematically stated as a nonlinear complementary problem (NCP) in the following: $\forall \alpha \in [\alpha^{\min}, \alpha^{\max}]$,

$$x_k^{wtm}(\alpha)\left[GC_k^{wtm}(\alpha, x) - \pi^{wtm}(\alpha, x)\right] = 0, \forall k \in K(w,t,m), w \in W, t \in T, m \in M \quad \text{(A5-a)}$$

$$GC_k^{wtm}(\alpha, x) - \pi^{wtm}(\alpha, x) \geq 0, \forall k \in K(w,t,m), w \in W, t \in T, m \in M \quad \text{(A5-b)}$$

$$\sum_{k \in K(w,t,m)} x_k^{wtm}(\alpha) = y^{wtm}(\alpha), \forall w \in W, t \in T, m \in M \quad \text{(A5-c)}$$

$$x_k^{wtm}(\alpha) \geq 0, \forall k \in K(w,t,m), w \in W, t \in T, m \in M \quad \text{(A5-d)}$$

where, $x = \{x_k^{wtm}(\alpha) \mid \forall k \in K(w,t,m), w \in W, t \in T, m \in M, \alpha \in [\alpha^{\min}, \alpha^{\max}]\}$ is a multi-class time-varying MDUE route flow vector, and $\pi^{wtm}(\alpha, x)$ is the time-varying minimum OD generalized travel cost for each mode, evaluated at x, for the trips with the same (w,t,m,α). Constraints in Eq. (1 A5-a,b) are complementary constraints. Constraints in Eq. (A5-c) are flow balance constraints. Constraints in Eq. (A5-d) are non-negative path flow constraints.

Given the assumptions and definition, the multi-class dynamic route choice model aims at solving the MDUE problem, under a given road pricing scheme, to obtain a time-varying route flow vector satisfying the MBDUE conditions. Based on the above NCP formulation, we can derive an equivalent gap function based nonlinear programming (NLP) formulation in Eq. (A6), which is an extension of an equivalent gap function based reformulation for the dynamic user equilibrium problem (A5).

$$\text{Min } g_R(x) = \sum_w \sum_t \sum_m \sum_k \int_{\alpha^{\min}}^{\alpha^{\max}} x_k^{wtm}(\alpha) \times \left[GC_k^{wtm}(x, \alpha) - \pi^{wtm}(x, \alpha)\right] d\alpha \quad \text{(A6)}$$

s.t. Eqs. (A5-b,c,d).

Integrated Multidimensional Network Choice Model

As shown in above two models, the integrated multidimensional network choice model essentially aims to seamlessly and correctly connect the mode choice model (TDMSUE) and multimodal dynamic route choice model (MDUE). This connection between these two models is the flow balance conditions as follows.

$$y_m^{wt} = \int_{\alpha^{\min}}^{\alpha^{\max}} y^{wtm}(\alpha) d\alpha = \int_{\alpha^{\min}}^{\alpha^{\max}} \left[\sum_{k \in K(w,t,m)} x_k^{wtm}(\alpha)\right] d\alpha, \forall w \in W, t \in T, m \in M \quad \text{(A7)}$$

Accordingly, the objective of the mode choice model is to obtain a TDMSUE mode flow pattern; and the objective of the multi-class dynamic route choice model is to obtain a MDUE route flow pattern with an input of the given TDMSUE mode flow pattern; in turn, the mode travel attributes (Level of Services) resulted for the MDUE route flow pattern lead to a new TDMSUE mode flow pattern. Therefore, the integrated multidimensional network choice model is mathematically formulated as follows.

Min $g_I(y,x) = \frac{1}{2} \times \sum_w \sum_t \sum_m \left[y_m^{wt} - q^{wt} \times p_m^{wt}(y\ ) \right]^2$ (A8)

s.t. Eqs. (A4-b,c, A5-a,b,c,d, and A8)

With the above integrated multidimensional network choice model formulation, the next section presents solution algorithms to solve this problem.

APPENDIX B5

Solution Algorithms for the Integrated Multidimensional Network Choice Model

5. Solution Algorithms for the Integrated Multidimensional Network Choice Model

This section presents solution algorithms and methods to the TDMSUE problem and MDUE problem.

Projected gradient based descent direction method to solve the TDMSUE problem

The TDMSUE problem can be decomposed into each OD pair and departure time. In light of Zhang et al. (2009), we can derive a cross-set gradient of the decomposed problem, $\nabla_{y_m^{wt}} g_M^{wt}(\mathbf{y})$, as follows.

$$\nabla_{y_m^{wt}} g_M^{wt}(\mathbf{y}) = y_m^{wt} - q^{wt} \times p_m^{wt}(\mathbf{y}) + \sum_{m' \in M} \left[y_{m'}^{wt} - q^{wt} \times p_{m'}^{wt}(\mathbf{y}) \right] \times \left[-q^{wt} \times \frac{\partial p_{m'}^{wt}(\mathbf{y})}{\partial y_m^{wt}} \right] \quad \text{(B1)}$$

Therefore, a projected gradient based descent direction based mode flow update scheme is as follows.

$$\mathbf{d}_y^{(n1)} = -\frac{\nabla_{\mathbf{y}} g(y^{(n1)})}{\left\| \nabla_{\mathbf{y}} g(\mathbf{y}^{(n1)}) \right\|} \times \mathbf{y}^{(n1)} \quad \text{(B2-a)}$$

$$\overline{\mathbf{d}}_y^{(n1)} = \prod_D \left(\overline{\mathbf{d}}_y^{(n1)} \right) = \left(\mathbf{I}_M - \mathbf{E}' \left(\mathbf{E}\mathbf{E}' \right)^{-1} \mathbf{E} \right) \times \mathbf{d}_y^{(n1)} \quad \text{(B2-b)}$$

$$\mathbf{y}^{(n1+1)} = \mathbf{y}^{(n1)} + \lambda^{(n1)} \times \overline{\mathbf{d}}_y^{(n1)} \quad \text{(B2-c)}$$

where, $\mathbf{d}_y^{(n1)}$ is the descent direction at iteration n1; $\overline{\mathbf{d}}_y^{(n1)}$ is the projected gradient based descent direction; $\left(\mathbf{I}_M - \mathbf{E}' \left(\mathbf{E}\mathbf{E}' \right)^{-1} \mathbf{E} \right)$ is the projection matrix. Eq. (B2-c) defines the mode flow update scheme from iteration n1 to iteration n1+1.

Parametric Analysis Method based Path Generation Including Reliability

The main impediment for solving the MDUE problem of interest is due largely to the relaxation of VOT from a constant to a continuous random variable and hence the need to find an equilibrium state resulting from the interactions of (possibly infinitely) many classes of trips, each of which corresponds to a class-specific VOT, in a network. If, in the extreme case, each trip-maker (or class) requires its own set of time-dependent least generalized cost paths, finding and storing such a grand path set is computationally intractable and memory intensive in (road) network applications of practical sizes. In order to circumvent the difficulty of finding and storing the least generalized cost path for each individual trip-maker with different VOT, the Parametric Analysis Method (PAM) is proposed to find the set of extreme efficient path trees, each of which minimizes the parametric path generalized cost function in Eq. (2) for a particular VOT subinterval (Mahmassani et al. 2005). The idea of finding the set of extreme efficient paths on which and heterogeneous trips are to be assigned is based on the assumption (See Dial 1996, Marcotte and Zhu 1997, Lu et al. 2008) that in the disutility minimization-based path choice modeling framework with convex disutility functions, all trips would choose only among the set of extreme efficient paths corresponding to the extreme points on the efficient frontier in the criterion space. Essentially, the parametric analysis method (PAM) based bi-criterion time-dependent least-cost path (BTDLCP) algorithm (Ziliaskopoulos and Mahmassani 1993) has two important roles to play in solving the proposed problem: (1) PAM based BTDLCP algorithm transforms a continuous distributed value of time into multiple user classes; (2) PAM based BTDLCP algorithm generates time-dependent least-cost path tree for each user class, which future defines a descent search direction for the MBDUE traffic assignment problem.

A novel approach is developed for this study to incorporate path-based reliability measures to obtain a multi-criterion generalized cost function as in Eq. (4) used in the underlying route choice model. The method leverages a powerful relation that is shown to hold between the standard deviation and the mean values of the travel time per unit distance. Combining this approach with the multiple paths produced by the parametric shortest path methods used to reflect heterogeneity in user preferences, a very efficient implement is devised in this work. Thus, for the given path set corresponding to the various classes of users (determined by the parametric shortest path procedure), the corresponding reliability measure is estimated directly at the path level, and re-labeling of the paths is then performed for the various classes taking reliability valuation into consideration.

In particular, for a given feasible route set of a triplet (w, t, m), the least experienced generalized cost route, will be different for different travelers, due to heterogeneous VOT. Following the same approach of Lu et al. (2008), the PAM can find a set of breakpoints, or values of α corresponding to the changes in the least experienced generalized cost path, that partition the feasible range of VOT α, $[\alpha^{\min}, \alpha^{\max}]$, into a set of subintervals, $\boldsymbol{\alpha}_{\mathbf{I}} = \{\alpha_0, \alpha_1, ..., \alpha_i, ..., \alpha_I \mid \alpha^{\min} = \alpha_0 < \alpha_1 < ... < \alpha_i < ... < \alpha_I = \alpha^{\max}\}$, and within each subinterval of VOT, $\alpha \in [\alpha_{i-1}, \alpha_i), \forall i = 1,..., I$, travelers are assumed to have the same path as their least experienced generalized cost path. For each α_i, OD pair w, departure time t, and mode m, let $\widetilde{K}(w,t,m,\alpha_i)$ to be a restricted route set.

Integrating time-varying and heterogeneous OD demand $y^{wtm}(\alpha)$ and route flow $x_k^{wtm}(\alpha)$ over each subinterval $[\alpha_{i-1}, \alpha_i), \forall i = 1,..., I$, we obtain the time-varying demand vector $y^{wtm}(\boldsymbol{\alpha}_{\mathbf{I}}) = \{y^{wtm}(\alpha_i) \mid \forall i = 1,..., I\}$, and route flow vector, $x_k^{wtm}(\boldsymbol{\alpha}_{\mathbf{I}}) = \{x_k^{wtm}(\alpha_i) \mid \forall i = 1,..., I\}$ for each user class i, as in Eq. (B3-a) and Eq. (B3-b), respectively.

$$y^{wtm}(\alpha_i) = \int_{\alpha_{i-1}}^{\alpha_i} y^{wtm}(\alpha) d\alpha = \int_{\alpha_{i-1}}^{\alpha_i} y^{wtm} \times \varphi(\alpha) d\alpha = y^{wtm} \times [\Phi(\alpha_i) - \Phi(\alpha_{i-1})] \quad \text{(B3-a)}$$

$$x_k^{wtm}(\alpha_i) = \int_{\alpha_{i-1}}^{\alpha_i} x_k^{wtm}(\alpha) d\alpha = \int_{\alpha_{i-1}}^{\alpha_i} x_k^{wtm} \times \varphi(\alpha) d\alpha = x_k^{wtm} \times [\Phi(\alpha_i) - \Phi(\alpha_{i-1})] \quad \text{(B3-b)}$$

Eq.(B3-a) and Eq.(B3-b) can be rewritten as Eq.(B4-a) and Eq.(B4-b), respectively.

$$y^{wtm} = \sum_{i=1}^{I} y^{wtm}(\alpha_i), \forall w \in W, t \in T, m \in M \quad \text{(B4-a)}$$

$$x_k^{wtm} = \sum_{i=1}^{I} x_k^{wtm}(\alpha_i), \forall k \in \widetilde{K}(w,t,m), w \in W, t \in T, m \in M \quad \text{(B4-b)}$$

Thus, the PAM procedure provides a way to transform the infinite-dimensional problem (continuous distributed VOT) to a finite-dimensional problem (multiple classes of VOT subintervals). Together with the set of restricted paths generated by the PAM procedure, a restricted multiclass dynamic user equilibrium (RMDUE) problem is formulated as follows.

$$\text{Min } g_R(x,\alpha_I) = \sum_w \sum_t \sum_m \sum_k \sum_i x_k^{wtm}(\alpha_i) \times \left[GC_k^{wtm}(x,\alpha_i) - \pi^{wtm}(x,\alpha_i)\right] \tag{B5-a}$$

s.t.

$$GC_k^{wtm}(x,\alpha_i) - \pi^{wtm}(x,\alpha_i) \geq 0, \forall k \in \widetilde{K}(w,t,m,\alpha_i), w \in W, t \in T, m \in M, i \in I \tag{B5-b}$$

$$\sum_{k \in \widetilde{K}(w,t,m,\alpha_i)} x_k^{wtm}(\alpha_i) = y^{wtm}(\alpha_i), \forall w \in W, t \in T, m \in M, i \in I \tag{B5-c}$$

$$x_k^{wtm}(\alpha_i) \geq 0, \forall k \in \widetilde{K}(w,t,m,\alpha_i), w \in W, t \in T, m \in M, i \in I \tag{B5-d}$$

Projected based Descent Direction Method to Solve the RMDUE Problem

The RMDUE problem can be decomposed into each OD pair and departure time. Based on Lu et al. (2008), we can derive a projected based descent direction method to solve the decomposed problem as follows.

$$d_k^{wtm,(n2)}(\alpha_i) = -1 \times \frac{GC_k^{wtm}(x,\alpha_i) - \pi^{wtm}(x,\alpha_i)}{GC_k^{wtm}(x,\alpha_i)} \tag{B6-a}$$

$$x_k^{wtm(n2+1)}(\alpha_i) = \begin{cases} Max\{0, x_k^{wtm(n2)}(\alpha_i) + \lambda^{(n2)} \times d_k^{wtm,(n2)}(\alpha_i)\}, \forall k \in K(w,t,m,\alpha_i) \setminus K_\pi(w,t,m,\alpha_i) \\ x_k^{wtm(n2)}(\alpha_i) + \dfrac{\sum_{k' \in K(w,t,m,\alpha_i) \setminus K_\pi(w,t,m,\alpha_i)} x_{k'}^{wtm(n2+1)}(\alpha_i)}{|K_\pi(w,t,m,\alpha_i)|}, \forall k \in K_\pi(w,t,m,\alpha_i) \end{cases} \tag{B7-b}$$

where, $d_k^{wtm,(n2)}(\alpha_i)$ is a descent direction of path k at iteration n2 for user class α_i and a triplet (w,t,m).

The next section presents a simulation-based column generation solution framework to solve the integrated model by using the above proposed methods and algorithms.

APPENDIX B6

Calibration of Time-Dependent OD Demand with Multiple Vehicle Types

6. Calibration of time-dependent OD demand with multiple vehicle types

This section presents a methodology to calibrate time-dependent OD demand with multiple vehicle types. Given static OD demand information and time-dependent link measurements, the dynamic OD demand estimation procedure aims to find a consistent time-dependent OD demand table that minimizes (1) the deviation between estimated link flows and observed link counts, and (2) the deviation between the estimated demand and the target demand (based on the static demand matrix). The induced network flow pattern can be expressed in terms of path flows and link flows.

Model Formulation and General Solution Procedure

In a dynamic context, and especially in congested networks, elements of the mapping matrix between OD demand and link flows are not constant and are, themselves, a function of the unknown OD demand values. By using a bi-level optimization algorithm, it is possible to obtain a TDOD matrix from a static Origin-Destination (OD) matrix and traffic count observations on specific links. The upper-level problem is an ordinary least-squares (OLS) problem, which is to estimate the TDOD demand based on given link-flow proportions. The link-flow proportions are in turn generated from the dynamic traffic network loading problem at the lower level, which may be solved by a simulation-based DTA procedure (in this case we use the DYNASMART-P (Jayakrishnan et al. 1994) software). The process is iterated until convergence in the reduction of root mean squared errors (RMSE) of the estimated link-flows is achieved (Mahmassani and Tavana 2001, Alibabai and Mahmassani 2008, Zhou et al. 2003).

Below is a list of parameters, variables and subscripts used in this section:

i: Subscript for origin zones, $i= 1, \ldots, I$.

I: Number of origin zones in the network.

i: Subscript for destination zones,$j = 1, \ldots, J$

J: Number of destination zones in the network.

g: Subscript for origin-destination pairs in the network captured by the lower-level problem, g = 1, …, G.

g: Set of origin-destination pairs in the network captured by the lower-level problem.

G: Number of origin-destination pairs in the network captured by the lower-level problem.

l: Subscript for count observation links,$l=1, \ldots, L$.

L: Number of links for which count observations exist.

$L1$: Number of links for which 15-minute count observations exist.

$L2$: Number of links for which 1-hour count observations exist, $L1+L2 = L$.

h : Subscript for “dynamic” (15-minute) departure times, $h = 1, \ldots, H$.

H: Number of “dynamic” (15-minute) departure times.

s: Subscript for “static” (1-hour) departure times, *s = 1, …, S.*

S: Number of “static” (1-hour) departure times.

t: Subscript for “dynamic” (15-minute) simulation/observation times, t = 1, …, T.

T : Number of “dynamic” (15-minute) simulation/observation times.

λ : Subscript for “static” (1-hour) simulation/observation times, $\lambda = 1,\ldots,\Lambda$

Λ : Number of “static” (1-hour) simulation/observation times.

c: Subscript for vehicle classes, *c = 1, …, C*.

C : Number of vehicle classes.

$M_{l,t}$: Simulated number of vehicles on link *l*, at “dynamic” time *t*.

$M_{l,\lambda}$: Simulated number of vehicles on link *l*, at “static” time λ .

$O_{l,t}$: Observed number of vehicles on link *l*, at “dynamic” time *t*.

$O_{l,\lambda}$: Observed number of vehicles on link *l*, at “static” time λ .

$d_{i,j,h}$: Generated demand for origin *i*, destination *j*, at departure time *h*.

$D_{g,h,c}$: Generated demand for origin-destination pair *g*, vehicle class *c*, at “dynamic” departure time *h*.

$\delta_{i,j}$: Target (given) demand for origin *i*, destination *j*.

$\Delta_{g,s,c}$: Target (given) demand for origin-destination pair *g*, vehicle class *c*, at “static” departure time *s*.

$p_{i,j,h,l,t}$: The proportion of demand for origin *i*, destination *j*, at departure time *h*, observed on link *l*, at simulation/observation time *t*.

$P_{g,h,l,t,c}$: The proportion of demand for origin-destination pair *g*, vehicle class *c*, at “dynamic” departure time *h,* observed on link *l*, at simulation/observation time *t*.

tw_t : Time weight for “dynamic” simulation/observation time *t*.

tw_λ : Time weight for “static” simulation/observation time λ .

lw_l : Link weight for count observation link *l*.

w: Weight on the deviation from the target demand for the unconstrained formulation,

$RMSE_{Flows}$: Root mean squared error for the difference between simulated and observed flows,

$RMSE_{Demand}$: Root mean squared error for the difference between target and generated demand.

In this study, for the New York Regional Network, one-hour demand tables are already available for the four-hour morning period to authors and are referred to as “static” departure times s, where the objective is to generate 15-min demand tables referred to as “dynamic” departure times *h*. Similarly, some traffic count observations are on 15-min time intervals referred to as “dynamic” observation times *t*, whereas some are on one-hour intervals referred to as “static” observation times λ.

Two objectives are considered in this model. The first one is to minimize the deviation between observed link flows and simulated link flows, and the second objective is to minimize the deviation between the static target demand and the estimated dynamic demand for two vehicle classes (SOV and HOV). From a multi-objective programming standpoint, the problem is transformed into a single-objective problem by a weighting formulation.

Basic Formulation

For the basic formulation, please refer to Cremer and Keller (1981). The first part of the objective function is trying to match the simulated flows $M_{l,t}$ with the observed flows $O_{l,t}$ by minimizing the squared deviations between them, whereas the second part of the objective tends to minimize the deviations between the generated demand and the target demand:

$$\min_{d_{i,j,h}}(1-w)\left(\sum_{l=1}^{L}\sum_{t=1}^{T} tw_t lw_l\left[M_{l,t}-O_{l,t}\right]^2\right)+(w)\left(\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\left\{\sum_{h=1}^{H} d_{i,j,h}\right\}-\delta_{i,j}\right]^2\right) \quad \text{(C1)}$$

$$\textbf{subject to}\; d_{i,j,h}\geq 0, \qquad \forall i,j,h$$

The simulated link flow is the sum of the products of link proportions and generated demand variables over OD pairs and departure times: $M_{l,t}=\sum_{i,j,h} p_{i,j,h,l,t} d_{i,j,h}$ With this conversion, the demand variables can be seen in both parts of the objective function:

$$\min_{D_{g,h,c}}(1-w)\left(\sum_{l=1}^{L}\sum_{t=1}^{T} tw_t lw_l\left[\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{h=1}^{H} p_{i,j,h,l,t} d_{i,j,h}\right\}-O_{l,t}\right]^2\right) + (w)\left(\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\left\{\sum_{h=1}^{H} d_{i,j,h}\right\}-\delta_{i,j}\right]^2\right) \quad \text{(C2)}$$

$$\textbf{subject to}\; d_{i,j,h}\geq 0, \qquad \forall i,j,h$$

Note that instead of dividing the total target demand into departure time intervals and forcing the generated demand at each departure time to match the divided target demand, the sum of the generated demand over time is aimed to match the static target demand. The first one would force a uniform distribution of demand, which would conflict with the purpose of creating a time-dependent OD table. Time weight tw_t and link weight lw_l are included to have the ability of differentiating the importance of some links and/or time intervals.

Modified Formulation

The formulation in Eq. (C3) is modified to meet the following requirements for large-scale networks:

- Reduction of variables by including only the OD pairs that are captured by the observation links in the lower-level problem, i.e. the ones that have non-zero link proportion values,
- Inclusion of different user classes,
- Utilizing the availability of hourly (or of other large interval size) demand matrices,
- Using observations of different time interval sizes.
- These are realized by making the following modifications:
- Creating a set **g**, which includes only the OD pairs that are captured by the lower-level problem (see Eq. C3),
- Making the lower-level problem solve the link proportions separately for each vehicle class c, and then summing the simulated flows over the classes in the upper-level problem to match the observations, if the observed counts are not separate for each vehicle class (see 1st and 2nd lines of Eq. C3),
- Summing the generated demand variable over departure times for each target demand of larger interval size separately (see 3rd line of Eq. C3),
- Making the lower level problem output the link proportions in the precision of the smallest observation interval available, and summing the simulated link flows over time to match larger interval sizes (see 2nd line of Eq. C3).

The modified formulation is given as follows:

$$\begin{aligned}\min_{D_{g,h,c}}(1-w)\Bigg(&\sum_{l=1}^{L1}\sum_{t=1}^{T} tw_t lw_l\left[\left\{\sum_{g=1}^{G}\sum_{h=1}^{H}\sum_{c=1}^{C} P_{g,h,l,t,c}D_{g,h,c}\right\}-O_{l,t}\right]^2\\&+\sum_{l=1}^{L2}\sum_{\lambda=1}^{\Lambda} tw_\lambda lw_l\left[\left\{\sum_{g=1}^{G}\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{t=4(\lambda-1)+1}^{4(\lambda-1)+4} P_{g,h,l,t,c}D_{g,h,c}\right\}-O_{l,\lambda}\right]^2\Bigg)\\&+(w)\left(\sum_{g=1}^{G}\sum_{s=1}^{S}\sum_{c=1}^{C}\left[\left\{\sum_{h=4(s-1)+1}^{4(s-1)+4} D_{g,h,c}\right\}-\Delta_{g,s,c}\right]^2\right)\end{aligned} \quad \text{(C3)}$$

$$\textbf{subject to } d_{i,j,h} \geq 0, \qquad \forall i,j,h$$

General solution procedure

Three types of input are required for solving this problem, namely the link proportions, link observations, and an initial estimate of the OD demand matrix (target matrix). DYNASMART-P is first run with the target matrix. Then the vehicle trajectory file (DYNASMART-P output) is post-processed to determine the link proportions. The OD estimation module is then executed and a more-consistent OD demand matrix is obtained and fed back into the system until convergence. The upper level problem can be solved by either an Interior/CG method using KNITRO 6.0.1 solver with AMPL language, a reduced gradient methods using MINOS 5.5 solver with GAMS language. The general procedure for solving this problem is depicted in Figure C1.

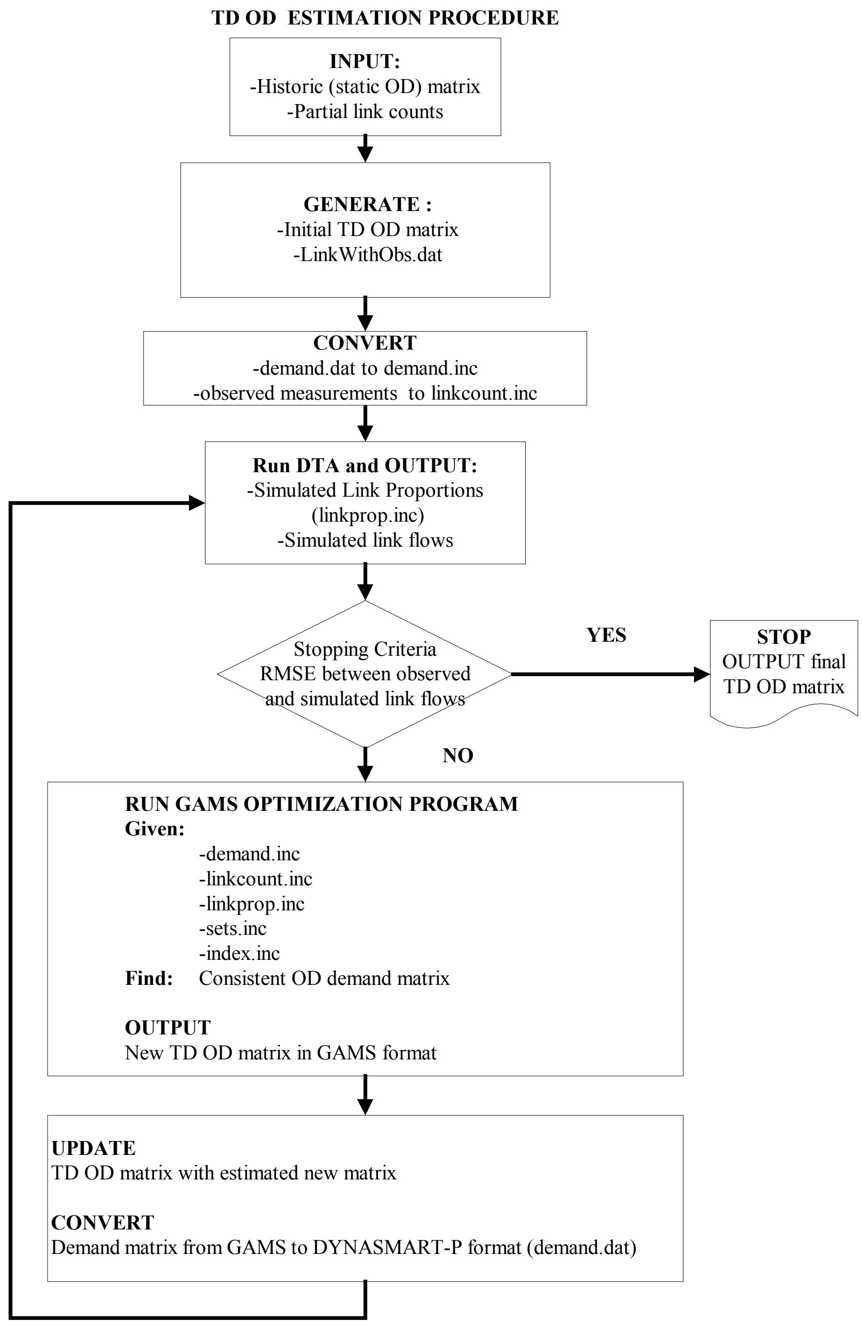


Figure C1. Schematic of the procedure for OD demand estimation

Challenges of Time-Dependent Origin-Destination (TDOD) Demand Estimation for Large-Scale Networks

Challenges for the upper level problem

As described in the previous section, the upper-level problem of TDOD estimation is an unconstrained ordinary least-squares (OLS) problem. In general, problems of that nature are easy to solve using today's nonlinear programming software tools. However, for large-scale networks such as New York region, the number of decision variables is of enormous size. The decision variables form an array of four dimensions:

- 3,697 origin zones,
- 3,697 destination zones,
- 16 departure time intervals of 15 minutes,
- 2 vehicle classes; single-occupancy and high-occupancy vehicles (SOV and HOV).

This array has about 4.37×10^{8} variables, which is either impossible or very time or memory demanding depending on the available software and computer configuration.

Challenges for the lower level problem

For the lower level problem, link proportions need to be solved for using the DTA procedure. Link proportions are an array of six dimensions:

- 3,697 origin zones,
- 3,697 destination zones,
- 16 departure time intervals of 15 minutes,
- 299 observation links,
- 20 observation time intervals of 15 minutes,
- 2 vehicle classes (SOV and HOV).

This array has about 2.62×10^{12} variables. The memory requirement to handle this amount of variables is around 11 terabytes (TB).

Methods to Overcome TDOD Estimation Challenges for Large-Scale Networks

Overcoming the challenges for the lower level problem

Since the lower level problem is solved first, the most logical way to overcome memory requirements is to reduce the number of variables. This can be established by sequential solving of link proportions. Since the link proportions are output to a file, there is no requirement to keep all the variables in memory.

Sequential solution of link proportions

By sequentially solving and outputting the values for an origin, clearing the array and doing it for the next origin, the array becomes smaller:

- 1 origin zone,
- 3,697 destination zones,

- 16 departure time intervals of 15 minutes,
- 299 observation links,
- 20 observation time intervals of 15 minutes,
- 2 vehicle classes (SOV and HOV).

This array has about 7.07×10^8 variables. The required memory is 3,697 times smaller than in the case of non-sequential solution: around 3 gigabytes. However, this methodology requires a lot of time. There are around 6.7 million vehicles traveling in the network and for each of the 3,697 origin zones, the code has to go through the trajectories of each vehicle.

Block-Sequential solution of link proportions

Since brute-force solution requires enormous memory and sequential solution requires a very long time, a logical approach is block-sequential solution. In this method, instead of going through the vehicle trajectories for a single origin zone at a time, the code goes through blocks of origin zones each time, solves the link proportions, clears the array and does these again for the next block:

- 7 origin zones,
- 3,697 destination zones,
- 16 departure time intervals of 15 minutes,
- 299 observation links,
- 20 observation time intervals of 15 minutes,
- 2 vehicle classes (SOV and HOV).

This array has about 4.95×10^9 variables. The required memory is around 528 times smaller than in the case of non-sequential solution and 7 times larger than in the case of sequential solution: around 21 gigabytes, which is a feasible requirement with the current configuration in hand. Furthermore, the calculation of link proportions may take up to one day, which is an acceptable time requirement for a network of this scale.

Overcoming the challenges for the upper level problem

As explained in section 0, there are around 437 million decision variables. The number of OD pairs is about 1.37×10^7. However, there are many OD pairs with zero demand. Depending on departure time, the number of OD pairs with non-zero demand is different; however, it is at most at around 2 million, which is a significantly less than 13 million. Furthermore, the vehicles using the observation links are from a limited number of OD pairs. The 299 observation links do not capture all the OD pairs:

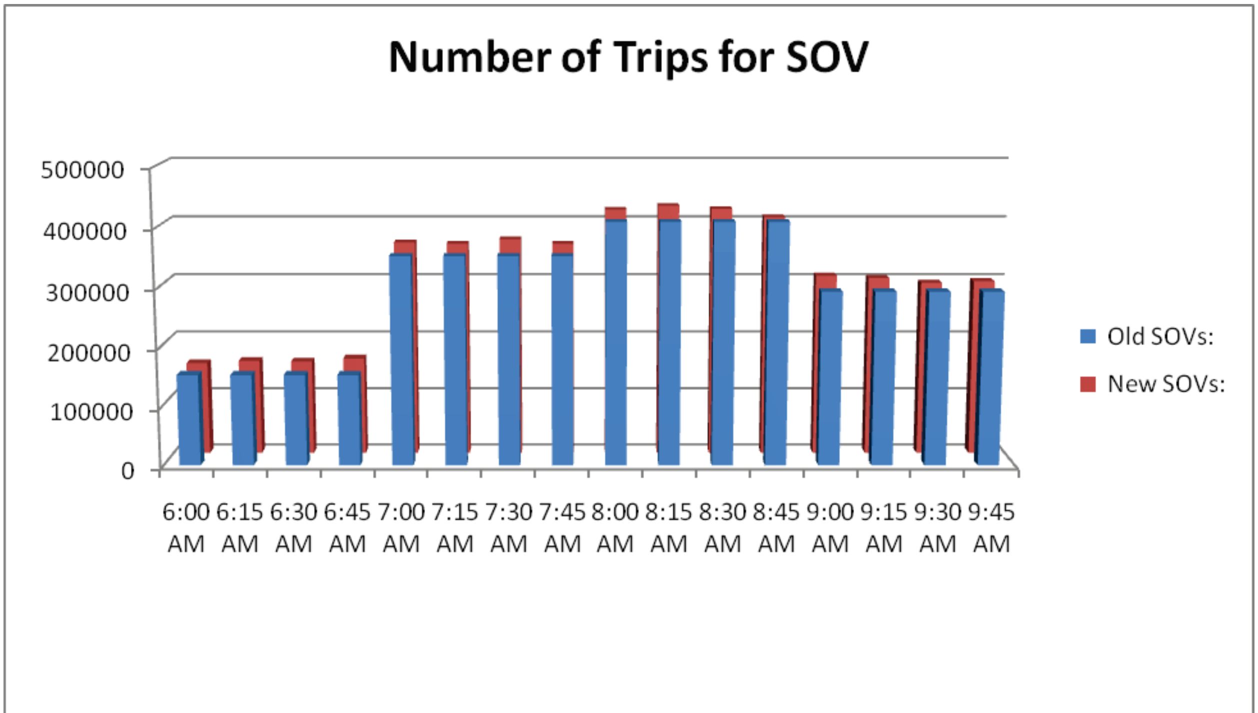
- ~300,000 OD pairs,
- 16 departure time intervals of 15 minutes,

- 2 vehicle classes (SOV and HOV).

This array has about 9.6×10^6 variables. Furthermore, these included OD pairs still have some zero values for some departure times and a vehicle class. Hence, the number of variables is further reduced to around 2 million. Reducing the number of decision variables from around 437 million to around 2 million is a significant improvement. This number of variables can be handled using today's optimization software on the computer configuration in hand. The solution takes several hours, which is again an acceptable duration for a large-scale network.

Generation of TDOD Demand and RMSE

Figure C2 depicts the time-dependent profile of the new demand files produced with the described procedure. The total LOV demand increased from 4.84 million to 4.97 million, and the total HOV from 1.88 million to 1.93 million as a result of the optimization problem. Figure C3 and Figure C4 illustrate the simulated and observed link volumes and cumulative volumes.



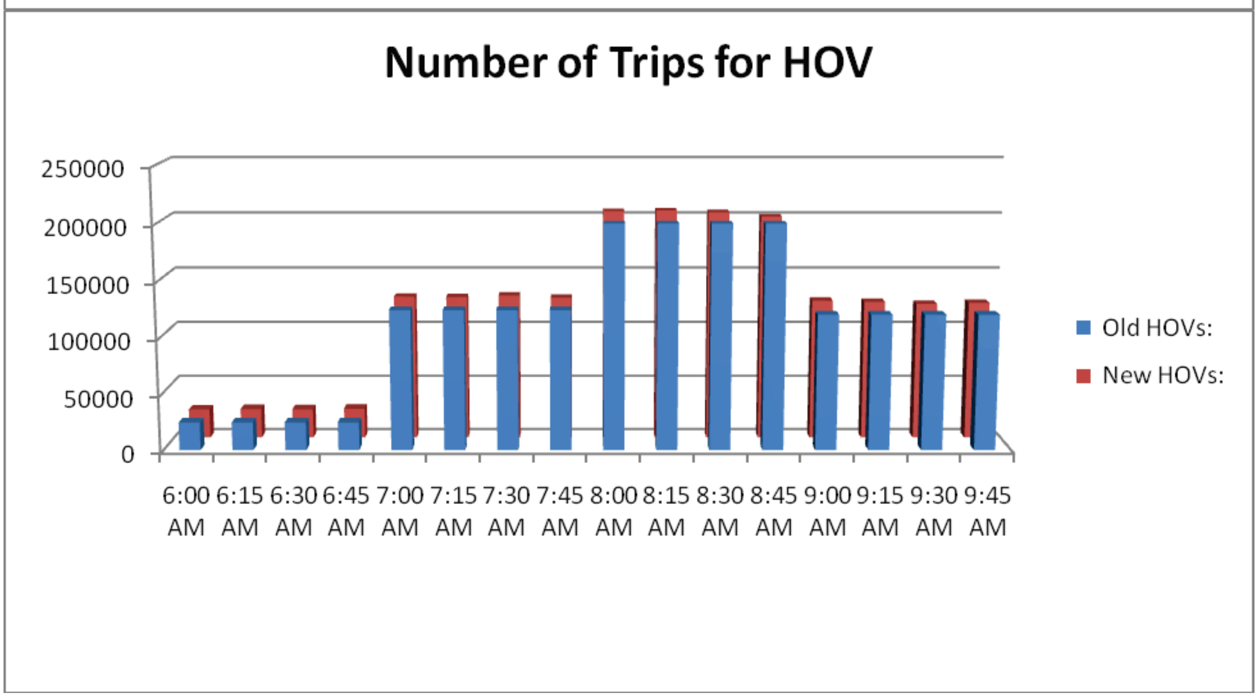


FIGURE C2 Total Number of Trips at each Departure Time

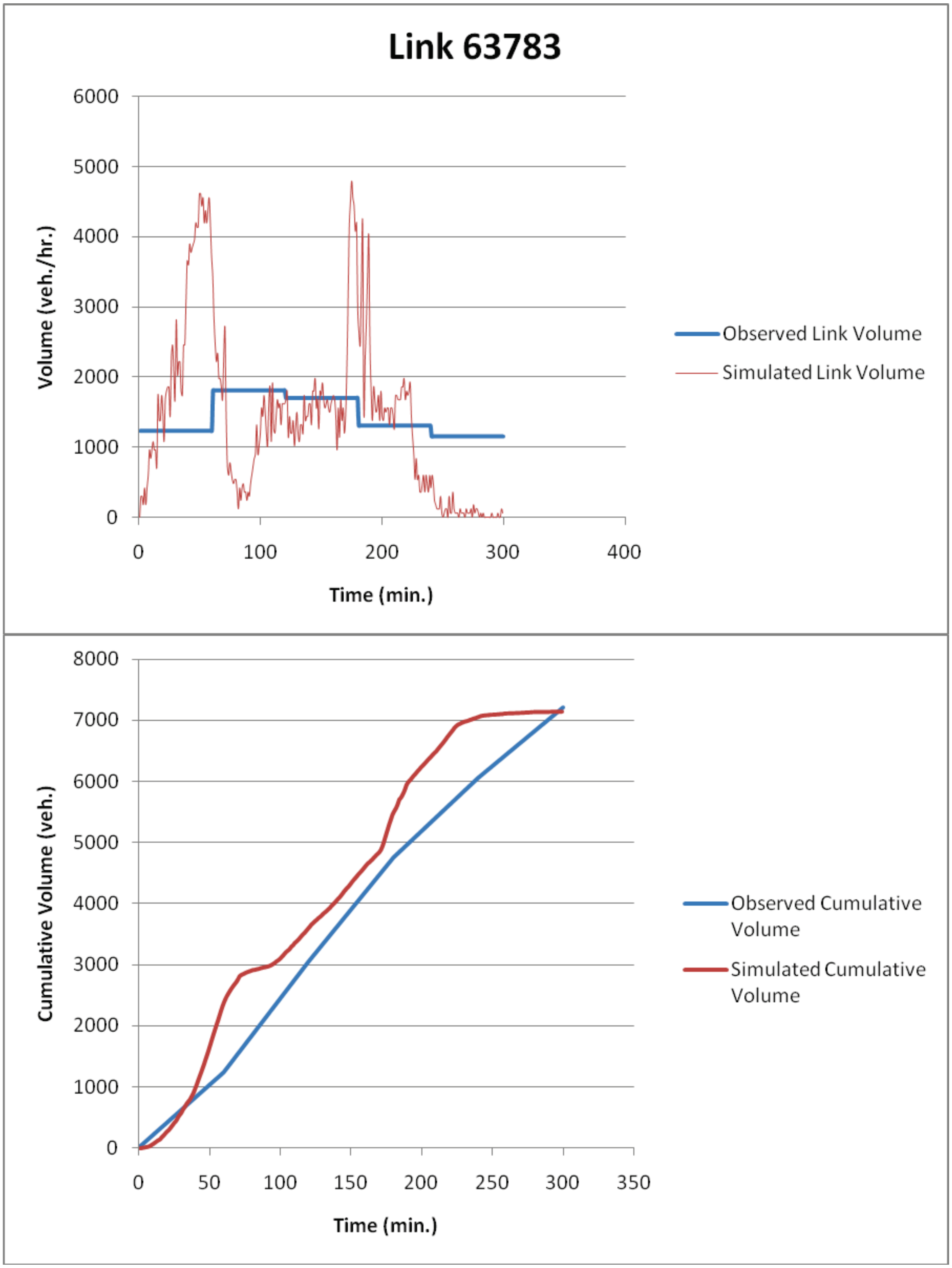


FIGURE C2 Connecticut Count Station 045, DIRECTION: EW, LOCATION: I-691, 0.5 MI WEST OF EB EXIT 3, AT PECK LA. UP, TOWN: CHESHIRE, 4/16/2008, 6-11 am

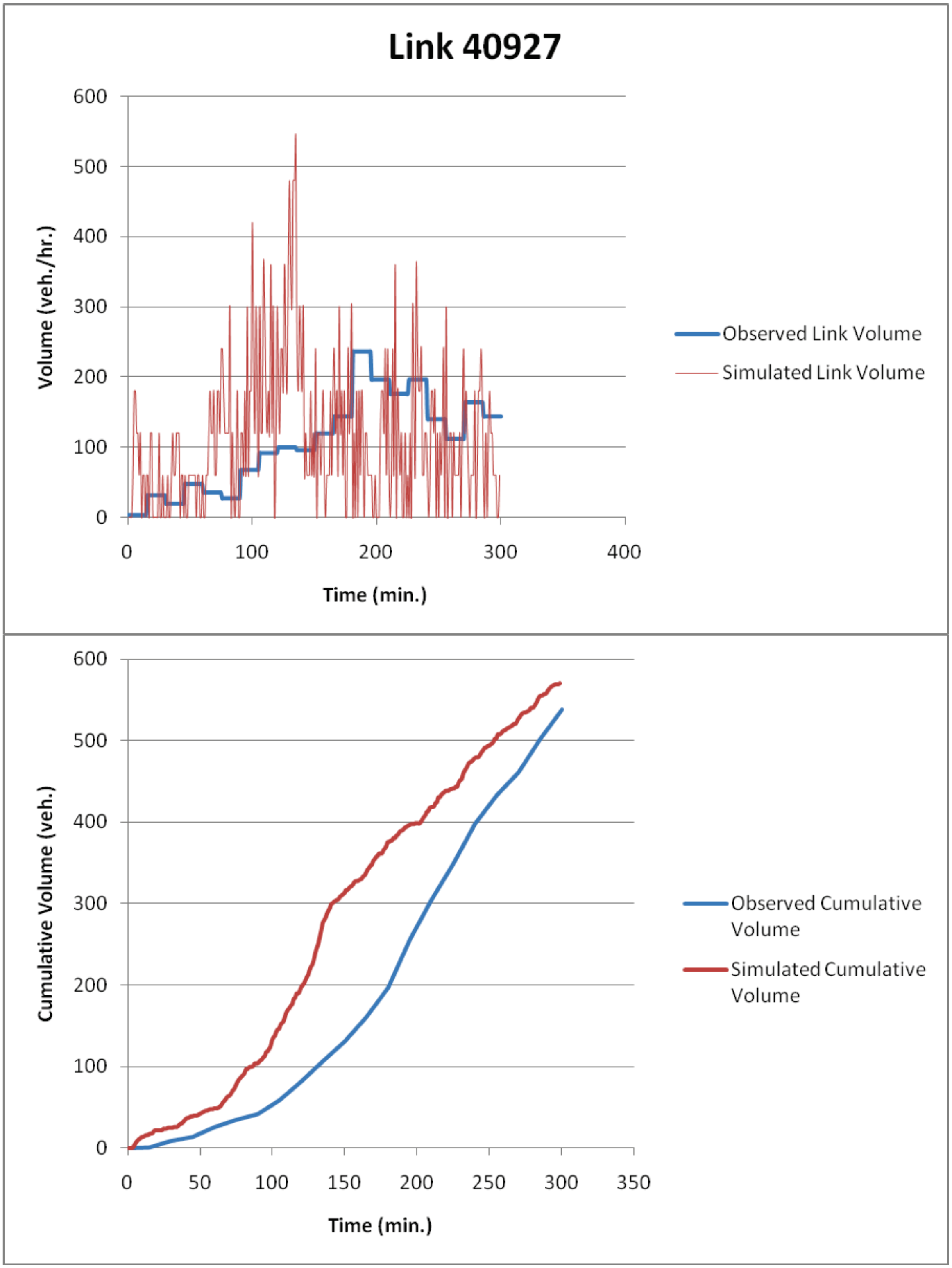


FIGURE C3 ROAD: EK10, ROAD NAME: FROST POND, RD FROM: BROOKVILLE LA, TO: PIPING ROCK RD, COUNTY: Nassau, 1/17/2008, 6-11 am

The final $RMSE_{Flows}$ is 14.391, which is a cardinal measure for the deviations between observed and simulated link flows. Compared to the first value, this corresponds to an improvement of 47.19 %. Since a simulation-based DTA is used in the lower-level, a reduction in every iteration cannot be guaranteed, because each time the DTA model is provided a new trip table, it will make different path choices for the trips. A further reduction in this value is expected after running further iterations of this estimation procedure. For a large-scale network, more observations are required in order to reduce the weight on demand in the multi-objective formulation and have a better improvement.

The increase in $RMSE_{Demand}$ is related to the increase in number of trips. However, the total number of trips seems to converge to an upper value of around 4.97 million, it does not increase indefinitely. Table C1 shows the change in demand and the root mean squared error over the iterations.

TABLE C1 Change in number of trips and RMSE values

	Number of Trips		RMSE Values	
	SOV	HOV	Demand	Flows
Original	4,844,509	1,881,843	0	27.252
Iteration 1	4,955,778	1,925,099	0.251	14.930
Iteration 2	4,964,743	1,929,071	0.262	15.679
Iteration 3	4,967,423	1,929,502	0.261	15.294
Iteration 4	4,967,745	1,928,512	0.259	14.391